

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B:
1. All question are compulsory.
 2. Figures to the right indicate full marks.
 3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1 Attempt **any three** of the following:

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- a. What do you mean by Universal Existential Statements & Existential Universal Statements with example.
- b. Construct an algebraic proof that for all sets P, Q, and R, $(P \cup Q) - R = (P - R) \cup (Q - R)$.
- c. Write notes on Russell's Paradox.
- d. Use truth tables to show the logical equivalence of the statement forms $p \vee q \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$.
- e. Define conjunction and Disjunction, construct its with truth table.
- f. Define tautology and contradiction. Show that the statement form $p \vee \neg p$ is a tautology and that the statement form $p \wedge \neg p$ is a contradiction.

Q.2 Attempt **any three** of the following:

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- a. Prove that $1+3\sqrt{2}$ is irrational.
- b. Suppose m is an integer. If $m \bmod 11 = 6$, what is $4m \bmod 11$?
- c. Consider the statement $\exists m \in \mathbb{Z}^+$ such that $m^2 = m$. Show that this statement is true and Let $E = \{5, 6, 7, 8\}$ and consider the statement $\exists m \in E$ such that $m^2 = m$. Show that this statement is false.
- d. Define Universal Quantifier and Existential Quantifier with example.
- e. Prove that "If an integer n is odd, then $5n-2$ is odd"?
- f. Show that the sum of any two rational numbers is rational.

Q.3 Attempt **any three** of the following:

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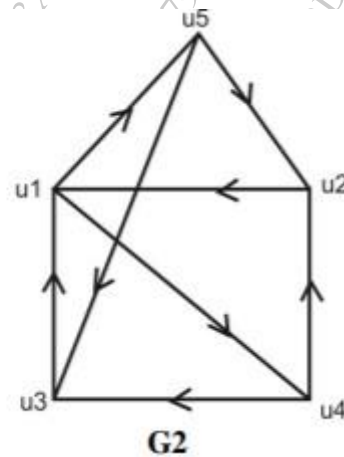
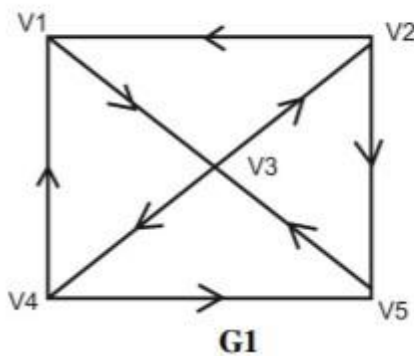
- a. Prove that, Prove that, $2^n < 5 + (n+1)!$ for all integers $n \geq 2$.
- b. Find the first four terms of each of the recursively defined sequences: $a_k = 2a_{k-1} + k$, for all integers $k \geq 2, a_1 = 1$

- c. Following sequences satisfies the given recurrence relation and initial conditions. Find an explicit formula for the sequence, $a_k = 2a_{k-1} + 3a_{k-2}$, for all integers $k \geq 2$, $a_0 = 1$, $a_1 = 2$.
- d. Define one-one and onto function with example.
- e. Let $X = \{1, 5, 9\}$ and $Y = \{3, 4, 7\}$. Define $f : X \rightarrow Y$ by specifying that. $f(1) = 4$, $f(5) = 7$, $f(9) = 4$. Is f one-to-one? Is f onto? Explain your answers.
- f. Define $g : Z \rightarrow Z$ by the rule $g(n) = 4n - 5$, for all integers n . Is g one-to-one? Prove or give a counter example.

Q.4 Attempt **any three** of the following:

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- a. Define subset, equal set, proper subset, null set, power set.
- b. State and explain properties of Binary Relation.
- c. Let $A = \{1, 2, 3\}$ and R be the relation defined on set A as $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$. Verify R is symmetric.
- d. Let $P = \{1, 2, 3, 6, 12\}$ and (P, \leq) is a partially ordered relation on relation \leq (less than and equal to). Show that it is linear or totally ordered relation. Also draw Hasse diagram.
- e. State and explain properties of tree.
- f. Define isomorphism of graphs. Check whether G_1 and G_2 are isomorphic or not.



Q.5 Attempt **any three** of the following:

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- a. State and explain Pigeonhole Principle
- b. Show that if any 29 people are selected then one may choose subset of 5. So that all 5 were born on the same day of the week.

- c. A bag contains 4 red marbles and 5 green marbles. Find the number of ways that 4 marbles can be selected from the bag, if selection contain i) No restriction of colors. ii) all are of same colors.
- d. State and explain any five types of events.
- e. A pair of fair dice is rolled. What is the probability that the sum of upper most face is 6, given that both of the numbers are odd?
- f. State and Explain Bayes theorem.
